

B.Sc. 1st Semester (Honours) Examination, 2022 (CBCS)

SH-1/PHSH/CC-1/23

Subject : Physics

Course : CC-I

Time: 2 Hours

Full Marks: 40

The figures in the margin indicate full marks.  
Candidates are required to give their answers in their own words as far as practicable.  
Abbreviations and symbols have their usual meaning.

**Group-A**

Answer any five questions.

2×5=10

1. (a) Describe the locus of a particle whose position vector is  $\vec{r}(\theta, \varphi) = (a \cos \theta \cos \varphi)\hat{i} + (b \sin \theta \cos \varphi)\hat{j} + (c \sin \varphi)\hat{k}$ .
- (b) Find the normal vector and equation of the tangent plane to the surface  $z = \sqrt{x^2 + y^2}$  at (3, 4, 5).
- (c) A water tank having height  $H$  and cross-sectional area  $A$  is completely filled. There is an orifice of cross-sectional area  $\alpha$  at the bottom. Calculate the time the tank will be empty when the orifice is opened.
- (d) Find the directional derivative of  $\Phi(x, y, z) = x^3 + y^3 + z^3$  at (1, -1, 2) in the direction of the vector  $\hat{i} + 2\hat{j} + \hat{k}$ .
- (e) Find the unit vector  $\hat{e}_r$  and  $\hat{e}_\theta$  in polar co-ordinate system.
- (f) Find the Taylor series expansion of  $e^x$  about  $x = 0$ .
- (g) Two players A and B play a game such that A has probability  $\frac{2}{3}$  of winning. If A plays 4 games, find the probability that A wins exactly 2 games.
- (h) Evaluate the integral  $\int (x^3 + 1) \delta(x^2 - 9) dx$ .

**Group-B**

Answer any two questions.

5×2=10

2. (a) Show that  $\int_C \frac{x dx + y dy}{\sqrt{x^2 + y^2}}$  is independent of the path of integration which does not pass through the origin. Find the value of the integral from the point  $P(-1, 2)$  to  $Q(2, 3)$ . 2+3
- (b) Solve the differential equation:  
$$\frac{dy}{dx} - y = y^2(\sin x + \cos x)$$
- (c) Find the expression of  $\vec{\nabla} \cdot \vec{V}$  in curvilinear co-ordinate system. Hence write the expression for spherical co-ordinates. 4+1

Please Turn Over

- (d) If  $f(x) = \frac{1}{2}(x + 1)$  for  $-1 < x < 1$  and  
 $= 0$  otherwise

is a density function, calculate  $E(x)$  and  $Var(x)$ .

2+3

### Group-C

Answer *any two* questions.

10×2=20

3. (a) Evaluate the integral  $\int_C [(x^2 - y^3)dx + (x + y)dy]$ , where  $C$  is the contour bounded by  $y = x$ ,  $y = -x$  and  $x^2 + y^2 = 16$  for  $x \geq 0$ .

- (b) Solve the differential equation:

5+5

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = xe^x \sin x$$

4. (a) A boat is rowed with a velocity  $u$  directly across a stream of width  $a$ . If the velocity of the current is directly proportional to the product of distances from the two banks, find the path of the boat and the distance where it will land.

- (b) Evaluate  $\iint \Phi \hat{n} ds$  where  $\Phi = \frac{3}{8}xyz$  and  $S$  is the surface of the cylinder  $x^2 + y^2 = 16$  included in the first octant between  $z = 0$  and  $z = 5$ .

5+5

5. (a) Represent the vector  $\vec{A} = 2y\hat{i} - z\hat{j} + 3x\hat{k}$  in spherical co-ordinates.

- (b) Prove that  $\int_{-\infty}^{\infty} f(x) \delta(x - a) dx = f(a)$ .

5+5

6. (a) Prove that Poisson distribution is the limiting case of binominal distribution for very large value of  $n$  (no. of trials) and small value of  $p$  (probability of occurrence) such that  $n.p$  is finite and equals to  $\lambda$ .

- (b) A set of observations is as:

$x:$  0 1 2 3 4

$f:$  30 63 45 10 2 where  $f = \text{frequency}$

Construct the distribution function. Hence calculate the theoretical frequency for the third observation ( $x = 2$ ). Given  $e^{-1.3} = 0.273$ .

6+2+2